Finite time singularities: from individual collapses to collapse turbulence

Pavel Lushnikov
University of New Mexico

Abstract

Many nonlinear systems have a striking phenomenon of spontaneous formation of singularities in a finite time (blow up). Blow up is often accompanied by a dramatic contraction of the spatial extent of solution, which has been called collapse since the pioneering work of Vladimir Zakharov in 1972. Near singularity point there is usually a qualitative change in underlying nonlinear phenomena, reduced models loose their applicability and other mechanisms become important such as inelastic collisions in the Bose-Einstein condensate; optical breakdown and dissipation in nonlinear optical media and plasma, wave breaking in hydrodynamics. Collapses occur in many physical and biological systems including a nonlinear Schroedinger equation (NLS), a Kadomtsev–Petviashvili equation with higher order nonlinearity, a Keller–Segel equation and many others. We will focus on NLS with dissipation and forcing in critical dimension. Without both linear and nonlinear dissipation NLS results in a finite-time singularity (collapse) for any initial conditions. Dissipation ensures collapse regularization. If dissipation is small then multiple near-singular collapses are randomly distributed in space and time forming collapse turbulence. Collapses are responsible for non-Gaussian tails in the probability distribution function of amplitude fluctuations which makes turbulence strong. Power law of non-Gaussian tails is obtained for strong NLS turbulence.