Modular Leonard Triples

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Let $K$ denote a field, and let $V$ denote a vector space over $K$ of finite positive dimension. An ordered triple $A, A^*, A^\circ$ of linear operators on $V$ is said to be a Leonard triple whenever for each $B \in \{A, A^*, A^\circ\}$, there exists a basis of $V$ with respect to which the matrix representing $B$ is diagonal and the matrices representing the other two operators are irreducible tridiagonal. A Leonard triple $A, A^*, A^\circ$ is said to be a modular whenever for each $B \in \{A, A^*, A^\circ\}$, there exists an antiautomorphism of $\text{End}(V)$ which fixes $B$ and swaps the other two operators. We present a characterization of the modular Leonard triples. This characterization involves explicit formulas for the entries of the matrices that represent $A, A^*$, and $A^\circ$ with respect to a particular basis. The formulas are expressed in terms of four algebraically independent parameters. We discuss how modular Leonard triples correspond to special Leonard pairs, and hence to particular discrete Askey-Wilson polynomials.