Starting from the classical Fejér-Jackson and Young inequalities for trigonometric sums we give several refinements, extensions and generalizations of them, and various related results. All of these are best understood in the context of more general orthogonal polynomials. In particular we extend and sharpen the Askey-Gasper inequality for sums of Jacobi polynomials. A related result for sums of Gegenbauer polynomials arising from function theory is also presented. Several applications of these inequalities are discussed.