Vietoris’ Theorem: A Complex View

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An analytic function \( f(z) \) in the unit disc \( \mathbb{D} \) is called \textit{stable} if \( s_n(f, \cdot)/f \prec 1/f \) holds for all for \( n \in \mathbb{N}_0 \). Here \( s_n \) stands for the \( n \)th partial sum of the Taylor expansion about the origin of \( f \), and \( \prec \) denotes the subordination of analytic functions in \( \mathbb{D} \). We prove that \( (1 - z)^{\lambda} \), \( \lambda \in [-1, 1] \) are stable. The stability of \( \sqrt{(1 + z)/(1 - z)} \) turns out to be equivalent to the famous result of Vietoris on non-negative trigonometric sums. We discuss some generalisations of these results, and related conjectures, always with an eye on applications to positivity results for trigonometric and other polynomials.

References


