QUIZ 3, MAT 460: VECTOR CALCULUS

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Name: ______________________________

(1) (1 point) Evaluate the determinant

$$\begin{vmatrix} a+1 & a & a & a \\ b & 2b & b & 2b \\ c & 2c & 3c & 4c \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

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(2) (1 point) Prove the identity $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}\times\mathbf{c})\times(\mathbf{a}+\mathbf{b})=0$.

(3) (2 points) Determine the value of the following limit

$$\lim_{(x,y)\to(0,0)}\frac{(1+x^2-y^2)\sin x}{x}.$$

(4) (2 points) Evaluate $\partial z/\partial x$ and $\partial z/\partial y$ if

$$z = \sqrt{e^{x+y^2} - xy^3}.$$

(5) (2 points) Find the differential of the following function

$$u = \frac{1}{\sqrt{x^4 + y^4 + z^4}}.$$

(6) (2 points) If $u = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, then show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2.$$

[Hint: use the chain rules to evaluate the derivatives on the right-hand side.]

(7) (2 points) If $z = f(\alpha x + \beta y)$, show that
$$\beta \frac{\partial z}{\partial x} - \alpha \frac{\partial z}{\partial y} = 0.$$

(8) (3 points) If $f(x, y)$ satisfies the identity $f(tx, ty) = t^n f(x, y)$ for a fixed n, show that
$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = n f(x, y).$$