QUIZ 4, MAT 460: VECTOR CALCULUS

Instructor: S. K. Suslov

Name: ______________________________

(1) (2 points) Prove that if $f(x, y)$ satisfies the Laplace equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

then so does

$$g(x, y) = f\left(\frac{x}{x^2 + y^2}, \frac{y}{x^2 + y^2}\right) \qquad \text{for } (x, y) \neq (0, 0).$$

Date: April 29, 2010.

(2) (2 points) Prove the identity $\operatorname{div}(\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot \operatorname{curl} \mathbf{A} - \mathbf{A} \cdot \operatorname{curl} \mathbf{B}$

(3) (2 points) Change the order of integration and evaluate

$$\int_0^2 \int_{y/2}^1 (x+y)^2 \ dxdy.$$

(4) (2 points) Evaluate the integral

$$\int\int\int_R \left(x^2+y^2+z^2\right)\ dxdydz.$$

Here R is the region bounded by $x+y+z=a>0$, $x=0$, $y=0$, and $z=0$.

(5) (2 points) Let $\mathbf{F} = 2yz\mathbf{i} + (-x + 3y + 2)\,\mathbf{j} + (x^2 + z)\,\mathbf{k}$. Evaluate $\int_S (\boldsymbol{\nabla} \times \mathbf{F}) \cdot d\mathbf{A}$, where S is the cylinder $x^2 + y^2 = a^2$, $0 \leq z \leq 1$ without the top and buttom.

(6) (2 points) Verify Green's theorem for the line integral

$$\int_C x^2 y \; dx + y \; dy$$

when C is the boundary of the region between the curves $y = x$ and $y = x^3$, $0 \leq x \leq 1$.

(7) (2 points) Evaluate the integral $\int_S \mathbf{F} \cdot d\mathbf{A}$ where $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + 3\mathbf{k}$ and S is the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.