(1) (20 points) If $x, y,$ and $b$ are greater then zero and $\frac{x}{y} < \frac{a}{b}$, prove that $\frac{x}{y} < \frac{x + a}{y + b} < \frac{a}{b}$. 
(2) (20 points) Prove that for all \( n \in \mathbb{N} \),
\[
1 + 3 + 5 + \ldots + (2n - 1) = n^2.
\]
[Hint: Use the Principle of Mathematical Induction.]

(3) (20 points) Suppose that \( x_n \) converges to 0. Show that \( 1/x_n \) is not bounded. [Hint: Use a proof by contradiction.]
(4) (10 points each) Evaluate the following limits when they exist

(a) \( \lim_{n \to \infty} \frac{n^3 + 12n^2 - n + 1}{2n^3 - 7n^2 + 2n - 12} \)

(b) \( \lim_{x \to \infty} \frac{x^7 - 2x^5 + 1}{x^7 + 5x - 1} \)

(c) \( \lim_{n \to \infty} \frac{(\cos n)^2}{n^2 + 1} \)
(5) (20 points) Define \( f : (0, 1) \to \mathbb{R} \) by \( f(x) = \frac{\sqrt{9-x} - 3}{x} \). Prove that \( f \) has a limit at 0 and find it.

(6) (20 points) Use the \( \varepsilon, \delta \) definition of the limit of function to show that \( \lim_{x \to 1} \frac{x^3 - 1}{x - 1} = 3 \). [Hint: \( x^3 - 1 = (x - 1)(x^2 + x + 1) \).]
(7) (20 points) Show that the sequence defined by $a_1 = 6$ and $a_n = \sqrt{6 + a_{n-1}}$ for $n > 1$ is convergent, and find the limit.
(8) Extra credit problem (5 points each)
(a) Define $f : (0, 1) \to \mathbb{R}$ by $f(x) = \sin(1/x)$. Does $f$ have a limit at $x = 0$? Justify.

(b) Define $f : (0, 1) \to \mathbb{R}$ by $f(x) = x \cos(1/x)$. Does $f$ have a limit at $x = 0$? Justify.