1 (5 points each) Evaluate

(a) $\begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix}^2$

(b) $a \cdot (a \times b)$

(c) $(a \times b) \cdot b$

(d) expand $(A - 2B)^2 C$, if $A, B$ and $C$ are $n \times n$ matrices
(2) (10 points) Find the first order partial derivatives of the following function

\[ f(x, y, z) = \frac{1}{(x^3 + y^3 + z^3)^{1/3}} \]

(3) (15 points) Find the second order partial derivatives \( u_{xx}, u_{xy}, u_{yx}, u_{yy} \) of the following function

\[ u = \ln \sqrt{x^2 + y^2} \]

and show that it satisfies the Laplace equation \( u_{xx} + u_{yy} = 0 \).
(4) (15 points) Consider the following function:
\[
    f(x, y) = \begin{cases} 
    \frac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\
    0, & (x, y) = (0, 0)
\end{cases}
\]
Show that \( f \) is not continuous at \( (0, 0) \). Do the derivatives \( \partial f / \partial x \) and \( \partial f / \partial y \) exist everywhere?

(5) (10 points) Let \( A = (a_{ik}) \) be a symmetric \( n \times n \) matrix (that is \( a_{ik} = a_{ki} \)) and define \( f(x) = x \cdot Ax \), so \( f : \mathbb{R}^n \to \mathbb{R} \). Show that \( \nabla f(x) = \partial f / \partial x = 2Ax \).
(6) (15 points) Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$z = \frac{u^2 + v^2}{u^2 - v^2}, \quad u = e^{x-y}, \quad v = e^{x y}.$$

(7) (15 points) The function from $\mathbb{R}^3$ to $\mathbb{R}^3$ that transforms spherical coordinates into Cartesian coordinates is given by

$$x = r \sin \theta \cos \varphi, \quad y = r \sin \theta \sin \varphi, \quad z = r \cos \theta.$$ 

Find the Jacobian

$$\frac{\partial (x, y, z)}{\partial (r, \theta, \varphi)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \varphi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \varphi} \end{vmatrix}.$$
(8) (Extra credit, 10 points) Suppose that $u(x,t)$ satisfies the differential equation $u_t + u u_x = 0$ and that $x$ as a function $x = f(t)$ of $t$, satisfies $dx/dt = u(x,t)$. Prove that $u(f(t),t)$ is a constant in $t$. 